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Discussion

Author's reply ☆

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Sanchez's paper [1] catches the attention of the asymptotic amplitude in my perturbation method [2], though the period obtained is of high accuracy. As it is well known that the perturbation solution is an asymptotic series, not a convergent one, which is an inherent shortcoming of perturbation techniques.

Consider the Duffing equation

$$\ddot{u} + u + \varepsilon u^3 = 0, \quad u(0) = A, \quad \dot{u}(0) = 0. \tag{1}$$

By the technique proposed in Ref. [2], we obtained the first-order approximate period (see Eq. 17 in Ref. [2])

$$T_1 = \frac{2\pi}{\sqrt{1 + \frac{3}{4}\varepsilon A^2}}. \tag{2}$$

The accuracy is about 7% when $\varepsilon \to \infty$.

The second-order approximate period (see Eq. (39) in Ref. [2]) reads

$$T_2 = \frac{2\pi}{\sqrt{\frac{1}{2}\left(1 + \frac{3}{4}\ \varepsilon A^2\right) + \frac{1}{2}\sqrt{1 + \frac{3}{2}\ \varepsilon A^2 + \frac{21\varepsilon^2 A^4}{32}}}}. \tag{3}$$

The 5.2% accuracy of the second-order approximation is much better than that of the first-order approximation when $\varepsilon \to \infty$. We have the very reason that the period converges fast to its

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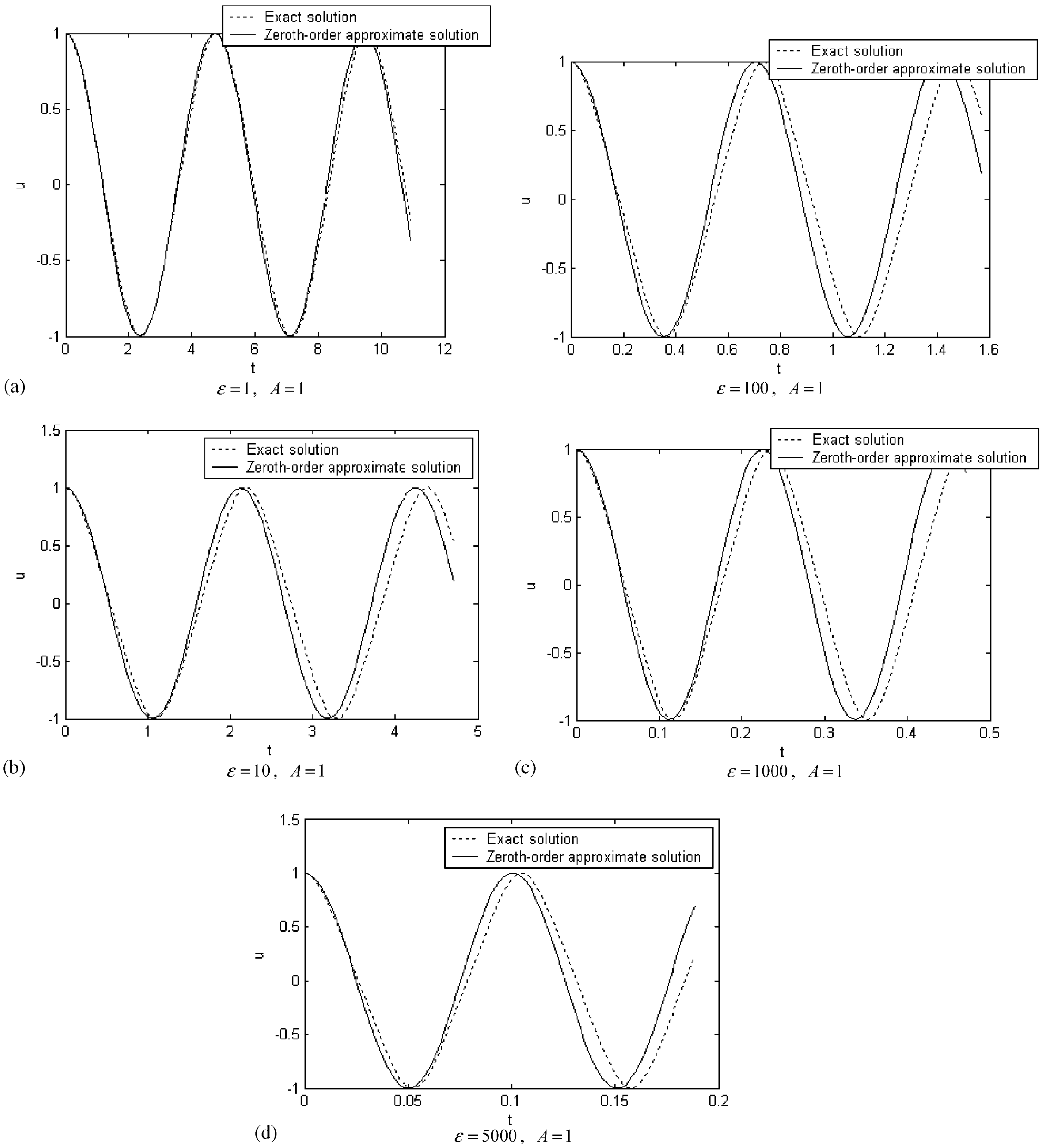

Fig. 1. Comparison of approximate solution with the approximate solution: (a) $\varepsilon = 1$, $A = 1$, (b) $\varepsilon = 10$, $A = 1$, (c) $\varepsilon = 100$, $A = 1$, $\varepsilon = 1000$, $A = 1$, (d) $\varepsilon = 5000$, $A = 1$.

exact solution. The remarkable advantage of technique proposed in Ref. [2] is that the obtained period is valid for the whole solution domain $0<\varepsilon<\infty$.

Our method is not intended to account for variation of the amplitude, for the amplitude does not vary with time for our discussed problem. How to deal with slowly varying amplitude, I have suggested an energy balance method [3,4]. Unfortunately the amplitude obtained by this technique is nonuniformly valid. As illustrated in Sanchez's paper, the amplitude becomes unbounded for large value of ε, this is the fact, but I disregard Sanchez's conclusion that my method is not an appropriate procedure and it leads to a complete wrong conclusion. We regard Sanchez's remark as a discussible hypothesis rather than an established fact.

The amplitude obtained by the method may diverge. However, if the series is an asymptotic expansion for amplitude, then for fixed n the first n terms in the expansion, $u = u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \cdots + \varepsilon^n u_n$, can represent the solution with an error that can be made small enough for all $0 < \varepsilon < \infty$. To our surprise, the fixed n equals to 0, that means the zeroth-order approximate solution is enough, no iteration is required. The zeroth-order approximate solution is

$$u = A \cos \left(1 + \frac{3}{4} \varepsilon A^2\right)^{1/2} t. \tag{4}$$

To illustrate the remarkable accuracy of my method, I compare my approximate solution with numerical solution for various cases from $\varepsilon = 1$ to $\varepsilon = 5000$, see Fig. 1.

If we want to search for a more high accurate solution, the frequency can updated as illustrated in Ref. [2], but we still keep one-term solution in the form

$$u(t) = A \cos \sqrt{\frac{1}{2}\left(1 + \frac{3}{4} \varepsilon A^2\right) + \frac{1}{2}\sqrt{1 + \frac{3}{2} \varepsilon A^2 + \frac{21\varepsilon^2 A^4}{32}}} t. \tag{5}$$

To summarize, I emphasize that the method is extremely simple, and is valid for both weak and strong nonlinear oscillators, the solutions obtained are valid for the whole solution domain. However, my method also has its limitation, the technique does not work if the amplitude of oscillation is a function of time. Some ways to deal with the problem can be found in Refs. [3,4], some advances in this method can be found in Refs. [5,6], applications of my method can be found in Refs. [7–12].

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